

## Welcome to BEAM's Challenge Set 4!

Want to keep doing fun and interesting math? Keep pushing yourself to learn new things? Each month, we're going to send you some fun math puzzles to try out.

Send in your solutions by:

- Texting photos to 424-305-6451,
- E-mailing them to [mathchallenges@beammath.org](mailto:mathchallenges@beammath.org), or
- Mailing them to us at the address above.

These problems are challenging, so anyone who sends correct solutions to 2 or more problems by the end of the day Tuesday, February 14 will get recognized in the Challenge Set solutions, and 4 or more wins a special prize. Your solutions may help for BEAM Pathway Program admission, too.

You might need to put some work on a separate page.

### Problem 1

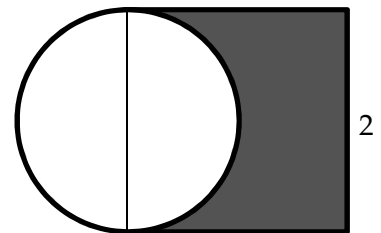
Here is a trick: a number is divisible by 9 if and only if the sum of its digits is divisible by 9. For example, say we want to know whether 2295 is divisible by 9. The sum of the digits is  $2 + 2 + 9 + 5 = 18$ . Because 18 is divisible by 9 ( $2 \times 9 = 18$ ), 2295 is also divisible by 9. On the other hand, 5934 is not divisible by 9 because  $5 + 9 + 3 + 4 = 21$ , which is not divisible by 9.

What is the smallest number that is divisible by 9 but has only 1s and 4s as digits?

### Problem 2

The figure on the right is a square with side length 2, and a circle that is placed so that one of the sides of the square goes exactly through the middle of the circle. What is the area of the region shaded in grey?

**Hint:** If you didn't get Problem 3 on Challenge Set 2, take a look at the solution before trying this one.



## Problem 3

Suppose that the symbols  $\Delta$  and  $\otimes$  mean this:

$$a \Delta b = 2 \times a - b \quad \text{and} \quad a \otimes b = \frac{a+b}{b}$$

For example:

$$3 \Delta 4 = 2 \times 3 - 4 = 2$$

$$10 \otimes 2 = \frac{10 + 2}{2} = 6$$

What is  $6 \otimes (5 \Delta 7)$ ?

## Problem 4

After a long and competitive game of Uno, Hector is down to one card. He's decided to give Giovanni and Aria a chance to guess it and tells them that his card is one of Red 4, Red 7, Blue 6, Green 3, Green 6, Yellow 3, Yellow 4, or Yellow 7.

Then Hector whispers only the color of his card to Giovanni and whispers only the number of his card to Aria. Giovanni and Aria then have the following conversation (and say nothing else):

Giovanni: I don't know what card Hector is holding.

Aria: I also don't know what card Hector is holding.

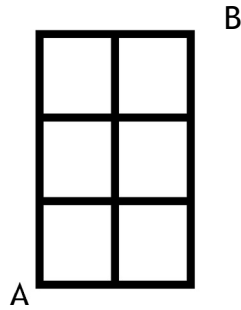
Giovanni: Then I know what card Hector is holding.

Aria: Then I also know what card Hector is holding!

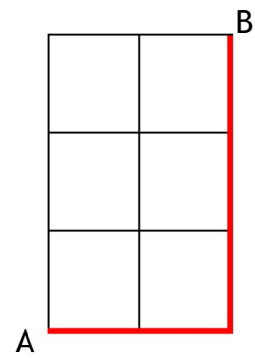
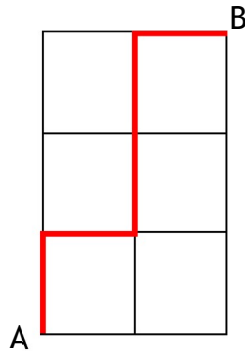
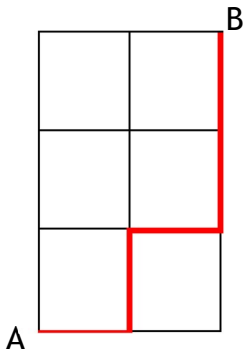
You can assume that Giovanni and Aria are telling the truth in this conversation, and that if there were a way to figure it out sooner, they would've known the card. What card is Hector holding?

## Problem 5

In the diagram below, there are many ways to get from A to B by following just the segments and moving only up and to the right.



For example, marked in red below are three paths that can get you from A to B:



There are many more paths like this. How many in total? Show or explain how you know.

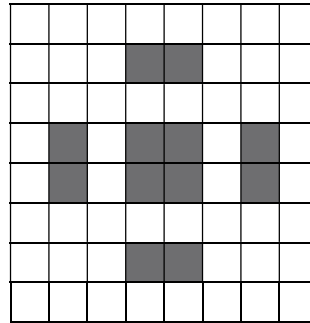
## Problem 6

Let's play the Game of Life, invented by John Conway, with a new starting design! We learned the rules of the game in Challenge Set 2 Problem 7, but let's review. Here's what happens during each round:

- Rule 1.** If a cell is alive and only has *0 or 1 living neighbors*, then it *dies*.
- Rule 2.** If a cell is alive and has 2 or 3 living neighbors, then it stays alive.
- Rule 3.** If a cell is alive and has *4 or more living neighbors*, it *dies* (it's overpopulated!).
- Rule 4.** If a dead cell has exactly 3 living neighbors, it comes alive from reproduction.  
 Otherwise, dead cells stay dead.

All of this happens at the same time each round!

Here's our new starting design:

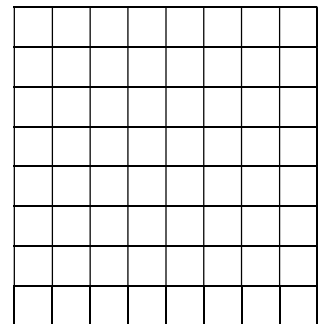
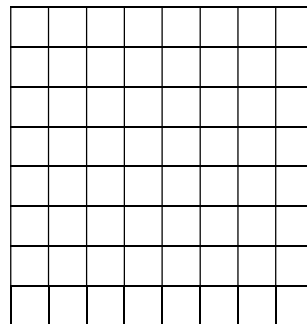
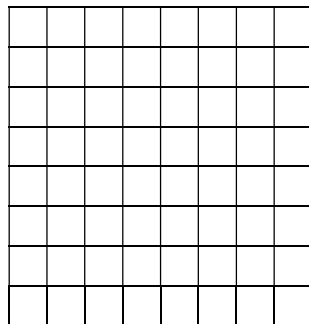
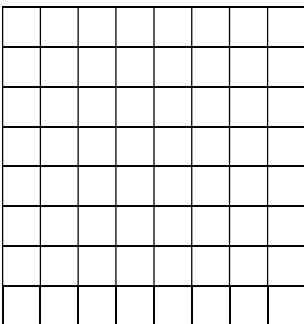


The gray cells represent living cells.

What does our design look like after 1 round? 2 rounds? 100 rounds? Draw each answer.

**Hint:** Try to solve Challenge Set 2 Problem 7 first and look at its solution.

If you need more space, try using graph paper! The grids below are meant to help you organize your thinking.



## Problem 7

Ashley, Nathan, and Sonia have decided to continue the prank from Challenge Set 3, Problem 5 on Carlos: each of them either always tells the truth or always lies for everything they say. They call someone a liar if they always lie and a truthteller if they always tell the truth.

Ashley says: "All of us are liars."

Nathan says: "Exactly one of us is a truthteller."

Is Sonia a liar or a truthteller? Explain why your answer must be correct and is the only correct answer.

*Problems 3 and 5 are originally from MATHCOUNTS. Problem 7 comes from "What is the name of this book?" by Raymond Smullyan.*



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