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BEAM Solvers

Congratulations to the successful solvers from Challenge Set 3!

Solvers (2+ Solutions) and Top Solvers (4+ Solutions, win a prize!)

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- Amelie Hurtado
- Aria Nahm
- Ashley Vincente
- Asuncion Gonzalez
- Benjamin Santos
- Diego Rojop
- Giovanni Baldovino
- Jailyn Mejia
- Jhony Gonzalez
- Jimena Perez Montano
- Jipper Reyes
- Kenny Gutierrez
- Matthew Servellon-Polanco
- Melanie Mendiola
- Michael Camarillo
- Nathan Echeverria
- Nehemias DeJesus
- Sean Cayetano
- Sonia Acabal
- Wilmer Roblero

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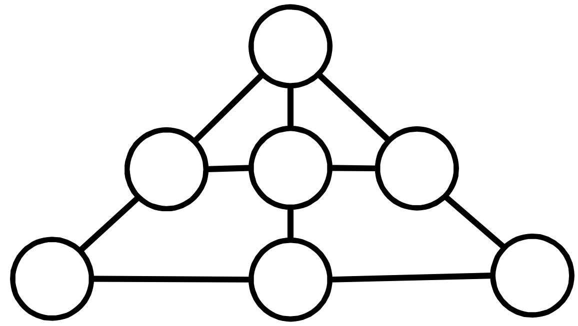
Challenge Set 3 Solutions

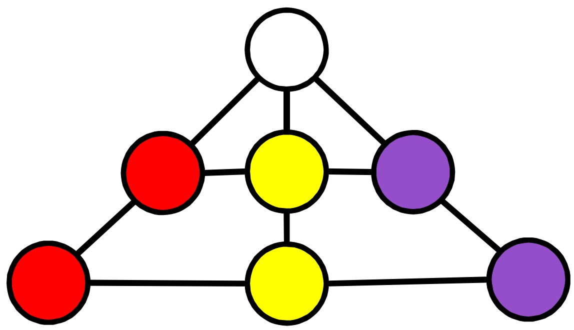
Problem 1

Place each of the numbers 1, 2, 3, 4, 5, 6, and 7 in the circles at right so that any three circles connected by a straight line add to the same sum. For example, if the bottom three circles have numbers that add to 3, then each other line of three circles must also add to 3.



Problem 1 Solution

Consider the red, yellow, and purple pairs of circles highlighted here:



Adding the white circle on the top to any two circles of the same color makes a line of three circles.

So we want to pair up numbers so that the following statement true:

red circles + white circle = yellow circles + white circle = purple circles + white circle.

Because we add the white circle to each sum, we only need the numbers in the colored circles to be equal. So we can simplify our problem further:

red circles = yellow circles = purple circles.

In general, if we want to create pairs that have the same sum, it makes sense to try to pair the largest and smallest numbers together. Doing so will help the sums "balance out."

Now let's think about the numbers we need to place in the circles:

1, 2, 3, 4, 5, 6, 7

Imagine we put a 7 in a red circle. Since 7 is the biggest number in the list, let's pair it up with the smallest; that means that the other red circle should have a 1 in it. We could put 1 and 7 in the red, 2 and 6 in the yellow, and 3 and 5 in the purple. This leaves only 4 to be put on the top circle.

Time to translate this reasoning into a solution!

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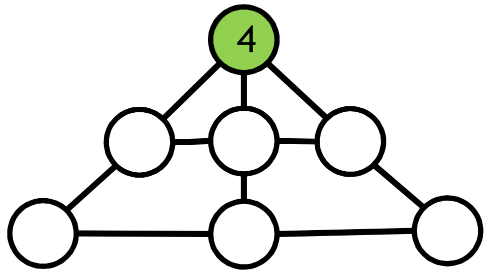
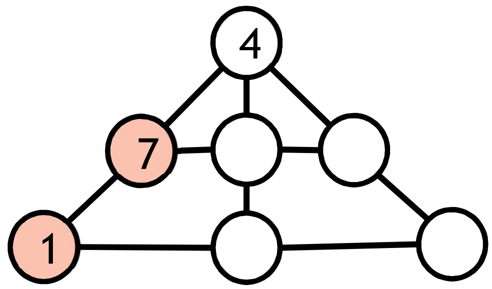
Let’s start with the 4 in the top circle:

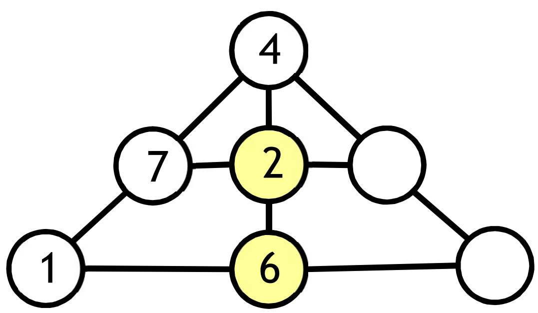


Let’s continue by placing 1 and 7 in the red circles – it doesn’t matter which one is on top and which on bottom:

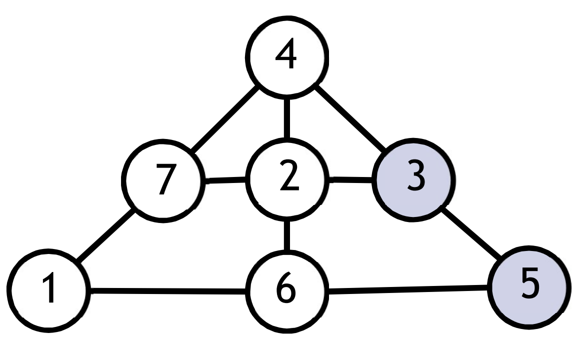


Now we have one entire line of three circles filled in! The sum of the line is $4 + 7 + 1 = 12$. That means we’ll want the sums along all the lines of three circles to be 12.

Next, let’s place 2 and 6 in the yellow circles. Our options for placing those numbers are a little more limited; if we place 6 in the same horizontal line as 7, the sum along that horizontal line will be too big (since $6 + 7 = 13$). So 6 has to be on bottom, like this:



Since $7 + 2 + 3 = 12$ and $1 + 6 + 5 = 12$, we’ll solve the problem by placing the last numbers like this:



At this point we should check that we didn’t make any mistake and that all lines really do add to 12. They do, so we’re done!

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Problem 2

Diana is running a board game activity. At the beginning of activities, every board game in the room is being played and there are three players at each game. Diana doesn't play because she is helping everyone. At the end of activities, Diana is also playing. This time, there are four players at each game, except for one game that no one is playing. How many board games are there in the room?

Problem 2 Solution

Solution 1: This problem can be solved with a careful guess-and-check. How, you ask?

- Since there are people playing each game at the end of activities, the total number of players (including Diana) is a multiple of four.
  Let's guess with multiples of 4.

- The number of student players (without Diana) also has to be divisible by 3, because there are 3 players per game at the start of activities.
  Seems like a good way to check guesses.

Could there be 4 players (including Diana)? Let's see.
At the start of activities, there are 3 student players (excluding Diana), and there must only be one board game. But at the end of activities, one game is left unplayed. That's impossible if there's only one board game, so there can't be 4 players total.

Could there be 8 players (including Diana)?
At the start of activities, there are 7 student players (excluding Diana). But we can't evenly divide up 7 players between board games so that each game has 3 players (there's always some players remaining!). That means 8 players is impossible, too.

We also run into the same issue if there are 12 players (including Diana): at the start of activities, there are 11 student players (excluding Diana), and we can't evenly divide 11 players evenly into groups of 3.

So could there be 16 players (including Diana)? I
n this situation, there would be 15 student players (without Diana) at the start of activities. Those 15 students could play 5 board games in groups 3 players each. Partway through activities, Diana joins and there are 16 players; 4 players each could play 4 board games, with one board game left over. This works!

We want to find the number of games in the room, so the answer is 5 board games total.

Solution 2: You could use algebra to solve the problem, too! Use the variable b (or whatever symbol you like) to represent the number of board games.

At the beginning of activities, we know there are 3 student players for every board game, plus Diana too, so the total number of players in the room is $3b + 1$.

At the end of activities, only $b - 1$ board games are played, and there are 4 players at each game. We could represent the number of players in either of the following ways:

$$4(b - 1)$$
$$4b - 4$$

The number of people in the room doesn't change, so these two expressions must be equivalent. Here's how we show that algebraically:

$$3b + 1 = 4b - 4$$

Add 4 to both sides to get:

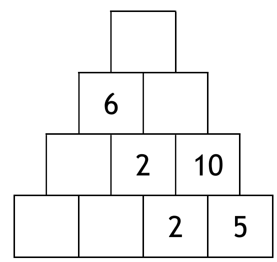
$$3b + 5 = 4b$$

Subtract $3b$ from both sides to get:

$$b = 5.$$

Problem 3

In the diagram on the right, the number in each square is the product of the two squares just below it. For example, since 2 x 5 = 10, 10 is in the square above 2 and 5. Fill in the rest of the squares.



Problem 3 Solution

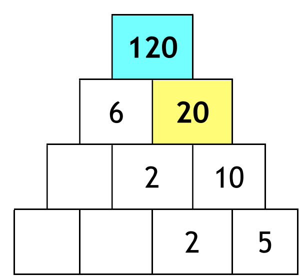
Like many math problems, if you go step-by-step you can solve this.

We'll show one order of filling in the boxes. You might have done it a different way and that's okay!

Let's start by filling in the yellow and blue boxes.

There is an empty box over a 2 and 10, and $2 \times 10 = 20$, so the empty box has to be 20.

Since $6 \times 20 = 120$, we can fill in the top box with 120.



Remember how the number pyramid from Challenge Set 1 (Problem 1) used addition, and so you had to subtract to go down the pyramid? Here, the number pyramid is based on multiplication, so you have to factor (a kind of division) to move down the pyramid.

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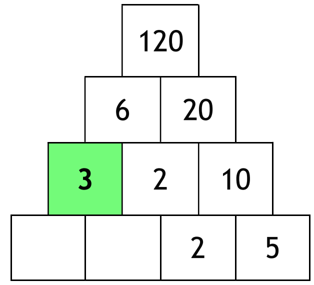
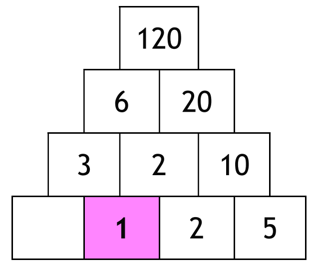
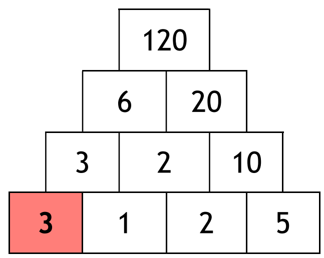
Next, the 6 is above a 2 and a blank box.
Since $2 \times 3 = 6$, that blank box must have a 3 in it.



Now, there's a blank box with a 2 above it and a 2 next to it. So that box must have a 1, since $2 \times 1 = 2$ (or, if you want to think of it as division, because $2 \div 2 = 1$).



The last blank box is next to a 1 and below a 3.
So it has to be a 3 too, since $1 \times 3 = 3$, or using division, because $3 \div 1 = 3$.



Problem 4
Go to the website below and watch the video

http://www.aops.com/videos/prealgebra/chapter10/197

Sean and Jimena are on a circular running track starting at the same place (point B). Sean starts running counterclockwise, and runs 4/5 of the way around the track. Jimena takes a shortcut and runs straight across the middle of the track, as in the diagram below. From the center of the track, what is the angle between Sean and Jimena now?

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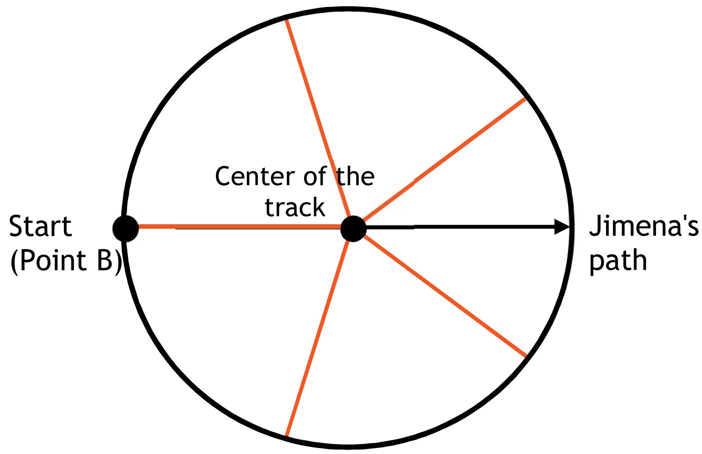
Problem 4 Solution

We want to see where Sean will be on the circle. We start by dividing the circle into five pieces:

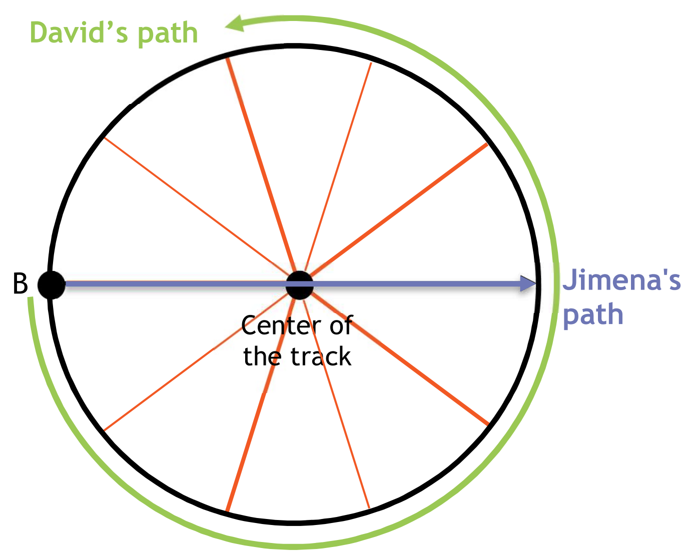
Since the entire circle is $360°$, 1/5 of the circle (in degrees) is

$$360° \times \frac{1}{5} = \frac{360°}{5} = 72°.$$



Sean ran $\frac{4}{5}$ of the way around counterclockwise, so he'll be at the 4th piece going around counterclockwise.

Solution 1: Jimena's path divides one of the $\frac{1}{5}$ pieces in half, so let's divide all of the $\frac{1}{5}$ pieces in half to make them easier to count:



Half of a $\frac{1}{5}$ piece is a $\frac{1}{10}$ piece, so each smaller piece is $\frac{1}{10}$ of the circle. Each $\frac{1}{10}$ piece is $36°$ since

$$72° \div 2 = 360° \times \frac{1}{10} = 36°.$$

Now we count that Sean and Jimena are three $\frac{1}{10}$ pieces apart. Since we showed that tenth of the circle is $36°$, the answer is $3 \times 36° = 108°$.

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Solution 2: Sean ran $\frac{4}{5}$ of the way around counterclockwise, and each fifth of the way round is $72°$, That means Sean travels $288°$ around the circle (because $72 \times 4 = 288$.)

By comparison, Jimena ends her trip exactly halfway around the circle. Half of $360°$ is $180°$.

To solve this problem, we can find the difference between Sean and Jimena's angles – the difference between their angles makes the distance between them. So $288° - 180° = 108°$.

Problem 5

On Carlos's birthday, BEAM Discovery students decide to pull a prank on her: some students will always lie to her, while other students will always tell the truth. Carlos walks up to Carolina and Michael, and asks Carolina: "Does at least one of the two of you always tell the truth?" Carolina says either "yes" or "no" in response to this. You don't hear what Carolina says.

Based on Carolina's answer, Carlos says she now knows whether Carolina always lies or always tells the truth and whether Michael always lies or always tells the truth. Does Michael always lie or always tell the truth? Explain why your answer is correct and why no other answers could be correct.

Problem 5 Solution

Let's call a student a trickster if they always lie and call a student a truthteller if they always tell the truth. There are only four possibilities for who is a trickster and who is a truthteller. Those possibilities are summarized in this chart:

	Carolina is a truthteller	**Carolina is a trickster**
Michael is a truthteller	Carolina: T, Michael: T	Carolina: L, Michael: T
Michael is a trickster	Carolina: T, Michael: L	Carolina: L, Michael: L

Let's look at all possible cases and figure out how Carolina would respond to Carlos's question.

Case 1: Carolina and Michael are both tricksters. In this case, the true answer to Carlos's question is no, but Carolina always lies, so he would answer yes.

Case 2: Carolina is a trickster and Michael is a truthteller. In this case, the true answer to Carlos's question is yes, but Carolina always lies, so he would answer no.

Case 3: Carolina is a truthteller and Michael is a truthteller. In this case, the true answer to Carlos's question is yes, and since Carolina always tells the truth, he would answer yes.

Case 4: Carolina is a truthteller and Michael is a trickster. In this case, the answer to Carlos's question is yes, and since Carolina always tells the truth, she would answer yes.

The problem says that as soon as Carolina answers the question, Carlos knows who lies and who tells the truth! If Carolina said yes, then Case 1, Case 3, and Case 4 are all possible and Carlos has no way of knowing which one it is. But if Carolina said no, then Carlos knows that Case 2 is true and that Carolina is a trickster and Michael is a truthteller. So Carlos will only have enough information to know everything in Case 2, and so we know that we are in Case 2. Thus, Michael is a truthteller.

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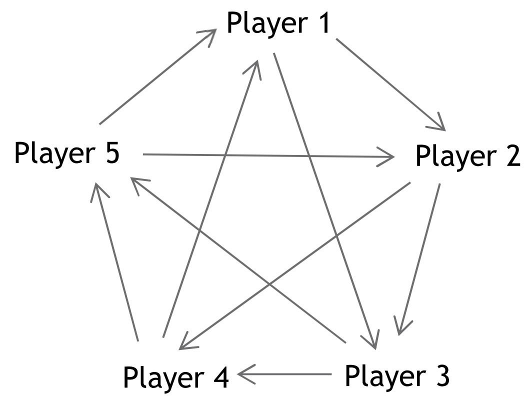
Problem 6

Five BEAM students decide to hold a chess tournament. Each student plays each of the other students exactly once. There are no ties, so every game ends with a winner. At the end of the tournament, some of the students tie for first place (most number of wins). What is the largest number of first place winners possible? Give one example of how that can happen.

Problem 6 Solution

There can be a 5-person tie. For example, one way this could happen:

- Player 1 wins against Player 2 and Player 3.
- Player 2 wins against Player 3 and Player 4.
- Player 3 wins against Player 4 and Player 5.
- Player 4 wins against Player 5 and Player 1.
- Player 5 wins against Player 1 and Player 2.



Sometimes it's helpful to use diagrams to draw out problems like this. In our diagram, arrows represent games, and the arrows point from a winner to a loser. (For example, the bottom arrow represents the game between Player 3 and Player 4. It shows that Player 3 wins because the arrow points toward the loser.)

Problem 7

A book has 30 stories. Each story takes up a different number of pages: one takes up 1 page, another takes up 2 pages, all the way up to the thirtieth story which takes up 30 pages. The first story starts on page 1. If the stories can go in any order, what is the greatest number of stories that can start on an odd page?

Problem 7 Solution

Let's start by thinking about a story with an even number of pages. If that story starts on an odd page, the next story will also start on an odd page. This is because an

odd number (starting page)	+	even number (number of pages in a story)	=	odd number (starting page of the next story)

If we start page 1 with a story with even pages, and we put all the even page stories at the start of the book, then all the even page stories will start on an odd page number. That's 15 stories that all start on an odd page already!

But is 15 the largest number of stories that start on odd pages? Let's keep thinking.

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Now let's think about the stories that have an odd number of pages.

When a story with an odd number of pages starts on an odd numbered page, the following story starts on an even page, because

odd number (starting page)	+	odd number (number of pages in a story)	=	even number (starting page of the next story)

On the other hand, when a story with an odd number of pages starts on an even numbered page, the following story will start on an even page, because

odd number (starting page)	+	even number (number of pages in a story)	=	odd number (starting page of the next story)

Let's put this all together. Once we're down to stories with only odd numbers of pages, they alternate which stories start on an odd page.

The first story with an odd number of pages starts on an odd page. The second starts on an even page. The third starts on an odd page. This pattern continues until the fifteenth story with an odd number of pages, which starts on an odd page. In total, that's 8 that start on an odd page number.

Thus, the total number of stories that start on an odd page will be $15 + 8 = 23$

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