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BEAM Solvers

Congratulations to the successful solvers from Challenge Set 2!

Solvers (2+ Solutions) and Top Solvers (4+ Solutions, win a prize!)

- Agustin Mendoza
- Alexander Lopez
- Alexander Rodriguez
- Aria Nahm
- Asuncion Gonzalez
- Benjamin Santos
- Caroline Corona
- Christopher Morales-Torres
- Diego Rojop
- Jacqueline Ornelas-Ventura
- Jailyn Mejia
- Jared Linares
- Jimena Perez Montano
- Jipper Reyes
- Katie Gomez
- Kimberly Perez-Bernal
- Lauren Pang-Le
- Maria Julia Allison Almario
- Matthew Servellon-Polanco
- Melody Zarate
- Michael Camarillo
- Nathan Echiverria
- Nehemias DeJesus
- Nohel Kim
- Sean Cayetano
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- Sonia Acabal
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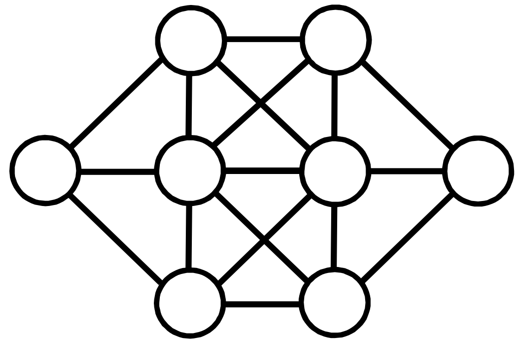
Challenge Set 2 Solutions

Problem 1

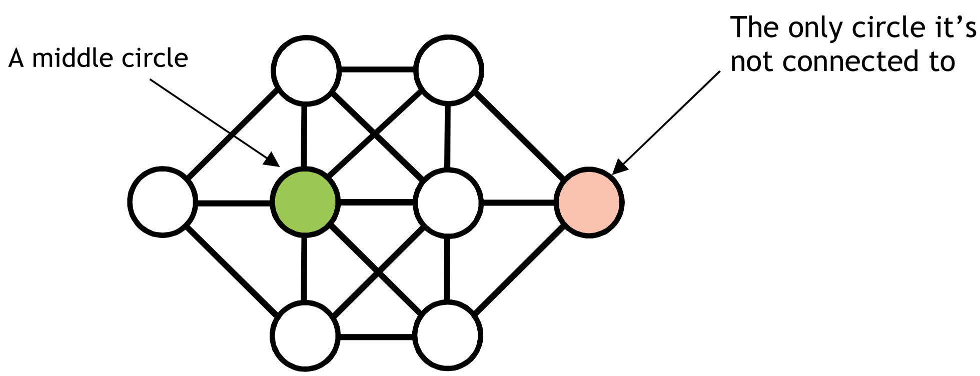
Place each of the numbers 1, 2, 3, 4, 5, 6, 7, and 8 in the circles at right so that no two numbers with a difference of one are in circles connected by a line.

For example, 2 and 3 cannot be in circles that are connected by a line because they are one apart.

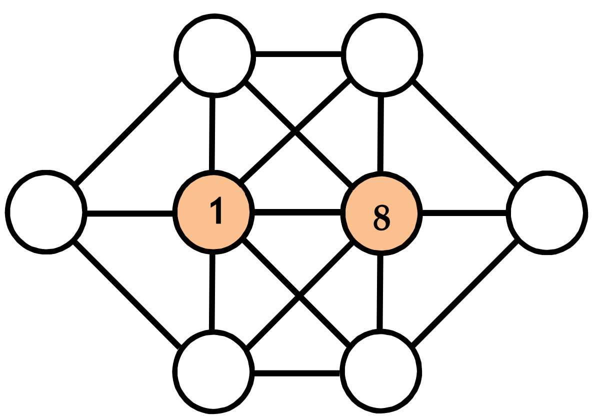


Problem 1 Solution

Trying out every possible solution would take much too long, but some thinking lets you see that 1 and 8 have to be in the middle circles. Why? Each of the middle circles is connected to six other circles, which means there is only one circle each middle circle is not connected to:



But that means 2 can't go in a middle circle, because 2 is one apart from 1 and 3, which means they'd both have to go into that one circle not connected to a middle circle. Something like that happens for every number except for 1 and 8 – they're only one apart from one other number! So the center circles have to be 1 and 8 or they could be 8 and 1. If we place them 1 and 8 it looks like this:



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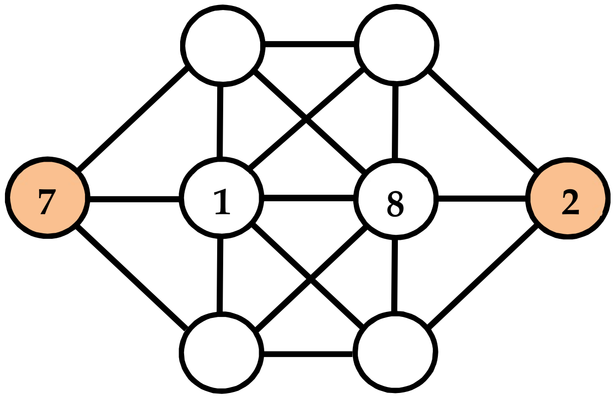
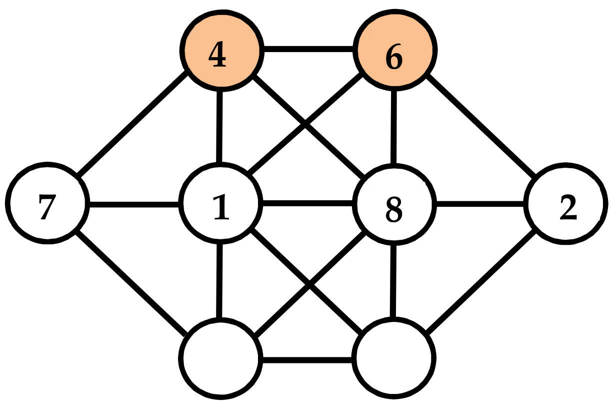
There is only one circle not connected to the circle with the 1, so that circle has to have the 2. In the same way, there's only one circle not connected to the circle with the 8, so that circle has to have the 7.

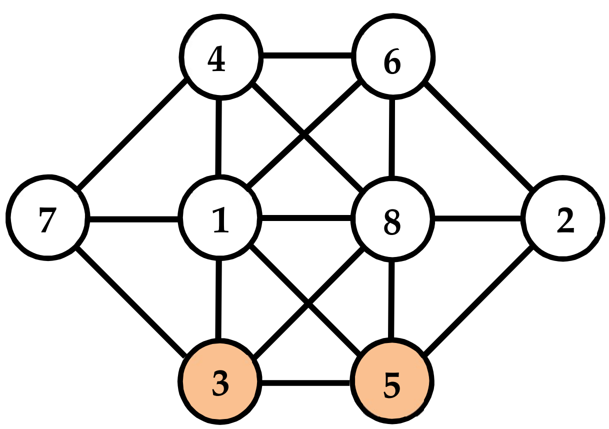


The two circles left on the top are connected to each other, and so are the two circles on the bottom. The numbers left are 3, 4, 5, and 6. If we consider 4, it can only be next to 6 but not next to 3 or 5. So if 4 is on top, then the other number on top has to be 6. But 6 can't be next to the 7, so that means it has to be on the right side. That would look like this:



Note that we could just as easily have put the 4 and 6 on the bottom as well and it would be the same, since they are connected to the other in the same way.

Now we only need to put the 3 and 5 in the last two circles. The 3 can't be next to the 2, so it has to be:



No guessing required!

Problem 2

Go to this website and watch the video on how to add every whole number between 1 and 100:

https://aops.com/videos/prealgebra/chapter1/19

Think about how the video does this, and use the strategy to add every number between 1 and 200:

$$1+2+3+\cdots+198+199+200=$$

Problem 2 Solution

In the video, we pair numbers so that every pair adds to 101 and there are 50 pairs, so we get

$$101\times 50=5050.$$

We do the same thing with this sum. When we pair 1 and 200, the outermost numbers, we get 201:

$$1+2+3+\cdots+198+199+200=$$

201

Similarly, when we pair the next inner-most numbers, the 2 and the 199, they also sum to 201.

201

$$1+2+3+\cdots+198+199+200=$$

201

This continues if we pair the numbers like this:

$$3+198=201$$

$$4+197=201$$

$$5+196=201$$

$$6+195=201$$

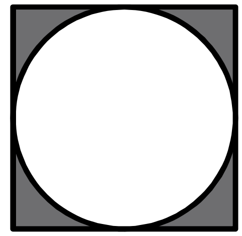
… and continuing on like that!

How many pairs are there? In the middle, the last numbers added are 100+101=201, so each number between 1 and 100 pairs with a number between 101 and 200. In total, each number 1–100 is part of one pair, so there are exactly 100 pairs. This becomes 100 201's, so the final sum (and answer to the problem) is

$$100\times 201=20100.$$

Problem 3

The picture on the right shows a square with side length 2 that has a circle removed from it.
What is the area of the shaded region?

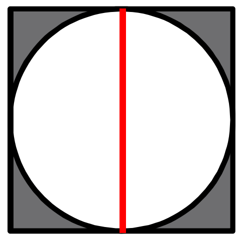


Hint: Remember that the area of a circle is $\pi \times r^2$.

Problem 3 Solution

The shaded region is the part that you get from taking the circle out of the square. Then the area of the shaded region is the area of the square minus the area of the circle. The area of the square is the side length multiplied by itself, so it's $2 \times 2 = 4$. To figure out the area of the circle, we first need to know the radius of the circle.

Consider the red line in this figure:



The red line splits the square in equal size rectangles, so it has the same length as the side of the square, which is 2. The red line is also a diameter of the circle, which means it is twice as long as the radius. So the radius of the circle has length 1.

Now we can plug that into the equation given in the problem. The area of the circle is:

$$\pi \times r^2 = \pi \times 1^2 = \pi.$$

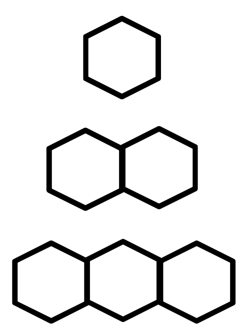
So the area of the shaded region is $4 - \pi$.

Note: Pi (π) is a special kind of number called an *Irrational number*. That means it can't be written as a decimal or fraction!

But what about 3.14, you might ask? That's a common approximation people use to calculate, but it's actually a different amount from pi. If you used 3.14 as an estimate for pi and got a final answer of 0.86, your answer is close, but technically not precise. We still gave credit for this answer, but it's always important to consider how exact your solutions need to be, and that depends on context.

Problem 4

Benjamin starts doodling in his notebook. First, he draws a hexagon, which has 6 sides. Then he draws two hexagons next to each other, and he counts 11 sides. Then he draws three hexagons next to each other, and he counts 16 sides. If he keeps going, how many sides will there be when he draws 50 hexagons next to each other?



As you learned this summer, you can often find the answer to a problem using a variety of solving strategies. This problem (and many of the Challenge Set problems) has many different ways of getting the same answer, and we want to share two of these ways with you.

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Problem 4 Solution

Solution 1: The first hexagon Benjamin draws has 6 sides. Every time he draws another hexagon connected to the end, he has to add 5 more sides. To get to 50 hexagons total, he adds 49 hexagons to his first hexagon, each of which will add 5 sides to the chain. This means the total number of sides are

$$6 + 5 \times 49 = 6 + 245 = 251,$$

where the 6 comes from the first hexagon, 5 stands for the five sides that make each extra hexagon, and 49 stands for the number of new hexagons he draws. Since this adds to 251, this is the answer.

Solution 2: Some students used algebraic expressions to find the number of sides in the diagrams. There were a few popular ways to build the expressions. We've written them below, along with what each part of each term represents. In each of the expressions, we've always used h to mean the number of hexagons.

$6 + 5(h - 1)$

- 6 represents the first hexagon
- 5 represents the additional sides needed for each new hexagon.
- (h – 1) represents the number of hexagons drawn next to the first hexagon.

$1 + 5h$

- 1 represents the first vertical line drawn.
- 5 represents the additional sides needed for each hexagon.

$6h - (h - 1)$

- 6 represents six sides drawn for each hexagon.
- (h – 1) counts the number of overlapping sides.

We can use properties of addition and multiplication (like the Commutative and/or the Associative Properties) to show that all three expressions are equivalent. That means once we choose the number of hexagons to draw in our design, any of the three expressions will calculate the exact same number of sides in the diagram, no matter what. And it would be a problem if the expressions were *not* equivalent – they're different ways to think about the exact same diagrams!

Problem 5

Sometimes, people mix up students' names and it's more than a little annoying. One BEAM student plays a game to point out how it's rude. They always use their real name on Thursdays and Fridays, use someone else's name on Tuesdays, and randomly uses their name or someone else's on other days of the week.

On seven consecutive days, the student was asked her name, and on the first six days she gave these answers in order:

Daisy, Emily, Daisy, Emily, Jacqueline, Emily

What answer(s) could she have given on the seventh day? A complete answer to this question must explain why no other answers are possible.

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Problem 5 Solution

We know the BEAM student always tells the truth on Thursdays and Fridays, so they'll have to say the same thing (her real name) twice in a row. The answers she gave are in order and she hasn't yet given the same name twice in a row. That means that she must repeat her real name on the 7th day. So her real name is either Emily or Daisy. In other words, either the last day is Thursday (and then the missing day is Friday, and she says Emily), or the first day is Friday (and the missing day was a Thursday, and she says Daisy). So her name must either be Emily or Daisy.

Case 1: The last day is Thursday. This means that she said her name was Emily on Thursday, and since she tells the truth on Thursdays, her name really is Emily.

Does this make sense with the rest of the days? If the last day is Thursday, the six days listed are:

Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
Daisy	Emily	Daisy	Emily	Jacqueline	Emily

But we also know that the BEAM student always lies on Tuesday, so we should check that she is lying on Tuesday. But by the above, she says her name is Emily on Tuesday. We already determined that her name must be Emily, so this isn't a lie! This is a contradiction; it cannot happen. So the last day cannot be Thursday.

Case 2: The first day is Friday. Since the name she gives on the first day is Daisy, and the problem tells us she always tells the truth on Friday, her name must be Daisy.

Does this make sense with the rest of the days? If Friday is the first day, then the days look like:

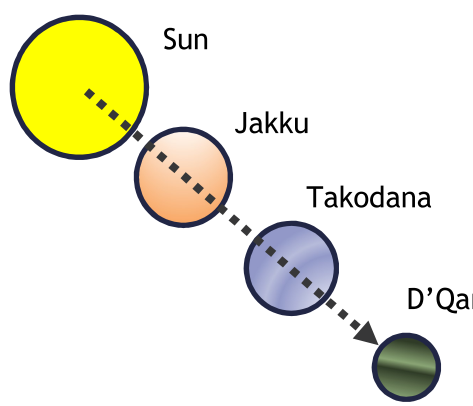
Friday	Saturday	Sunday	Monday	Tuesday	Wednesday
Daisy	Emily	Daisy	Emily	Jacqueline	Emily

We know she always tells the truth on Thursdays and Fridays, so if this case works she must be Daisy. Does it match the other parts of the problem? The problem says she lies on Tuesdays. For this case, on Tuesday she says she's Jacqueline, which would be a lie. Since she can either lie or tell the truth on the other days, this is consistent and so her name really is Daisy.

Since the next day is Thursday, she tells the truth the next day, and the answer to the problem is Daisy.

Problem 6

The Star Wars planets Jakku, Takodana, and D'Qar all orbit the same sun. Jakku takes 250 days to orbit all the way around to the same place, Takodana takes 300 days, and D'Qar takes 360 days. If the three planets are lined up in a line including the Sun, as shown, what is the minimum positive number of days before they are all in the exact same locations again?



Problem 6 Solutions

This problem also has multiple solutions we want to share with you!

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Solution 1:
If it takes Jakku 250 days to orbit all the way around to the same place, then it will be at the same position again only at multiples of 250, that is: 250, 500, 750, … This is also true about Takodana, but for multiples of 300, and D'Qar, but for multiples of 360. So the number we want is a multiple of 250, 300, and 360. Since we want the minimum number, this means we are looking for the least common multiple of 250, 300, 360.

Solution 2:
Here's an elegant way of solving the problem. Some students solved a related problem with friendlier numbers, then used their work to solve the original problem. Solving similar problems Is a great strategy when you're stuck (or don't want to work with huge / tiny / unruly numbers!)

Some students noticed that 250 (Jakku's orbit), 300 (Takodana's orbit), and 360 (D'Qar's orbit) are all multiples of 10. Instead of thinking about Jakku, Takonda, and D'Qar, they explored what would happen to planets with 25-day, 30-day, and 36-day orbits. By listing out the multiples of 25, 30, and 36, they saw that the least common multiple of those numbers was 900.

Once the students found the LCM of 25, 30, and 36, they applied their findings to the original problem. The orbits in the smaller problem were all $1/10^{th}$ of the original problem's orbits; that means the LCM of the values in the original problem is 10 times greater than 900. Again, 9,000!

Solution 3:
Lots of people calculated the least common multiple by listing the multiples of the numbers, but here's a trick: we can use primes! The least common multiple must be divisible by all the same primes as the numbers we started with, and no more. Factoring our original values will help identify factors for our LCM:

$$250 = 2 \times 5 \times 5 \times 5$$

$$300 = 2 \times 2 \times 3 \times 5 \times 5$$

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

(You can also write these factorizations using exponents, as 2×5^3, $2^2 \times 3 \times 5^2$, and $2^3 \times 3^2 \times 5$.)

Any number that is a multiple of 250 must be divisible by 2 and by three 5's. Any number that is a multiple of 300 must be divisible by two 2's, a 3, and two 5's. Any number that is divisible by 360 must be divisible by three 2's, two 3's, and a 5.

If a number that is divisible by at least three 2's, two 3's, and three 5's, then that number will be a multiple of all three numbers at once. To find the *least* common multiple, multiply only these factors together (with nothing else):

$$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 = 9000.$$

(Hint: the quick way to do this is to see that $2 \times 5 = 10$, so you can simplify to $3 \times 3 \times 10 \times 10 \times 10$.)

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Solution 4:
Some students used tables to efficiently factor 250, 300, and 360. On the left side of the table, they wrote a factor. Then, inside the table, they factored 250, 300, and 360 by the number on the left, all at once. They wrote the results as a new row in the table. For example, students started by writing 10 on the left. Then, they wrote 25, 30, and 36 in the table, since $250 \div 10 = 25$, $300 \div 10 = 30$, and $360 \div 10 = 36$. If a value on the left did not factor a number in the table, they kept that value as-is. Here's what their tables looked like:

	250	300	360
10	25	30	36
5	5	6	36
6	5	1	6
	5	1	6

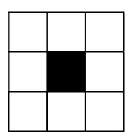
They kept factoring until the three resulting values had no shared factors.

Last, they found the product of the factors along the outside of the table. That product, worth $10 \times 5 \times 6 \times 5 \times 6$, is 9,000. That means 9,000 days is the time until the next planet alignment.

Problem 7

In the Game of Life, invented by the mathematician John Conway, cells live and die based on how many of their neighbors are alive.

Here's how it works. The small squares below can be called "cells". Each cell has eight neighbors. In the picture below, 8 cells are *dead* and 1 cell is *alive*. Which is which?

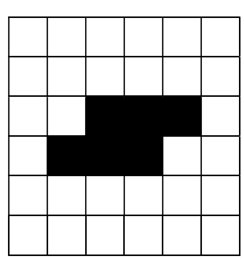


The game is played in rounds. Each round cells can live, *die,* or come alive, based on these rules:

Rule 1. If a cell is alive and only has *0 or 1 living neighbors*, then it *dies*.
Rule 2. If a cell is alive and has 2 or 3 living neighbors, then it stays alive.
Rule 3. If a cell is alive and has *4 or more living neighbors*, it *dies* (it's overpopulated!).
Rule 4. If a dead cell has exactly 3 living neighbors, it comes alive from reproduction. Otherwise, dead cells stay dead.

All of this happens at the same time each round!

Now, suppose we start with this grid:



What does it look like after 1 round? After 3 rounds? After 50 rounds?

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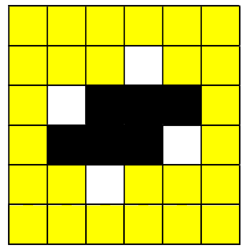
Problem 7 Solution

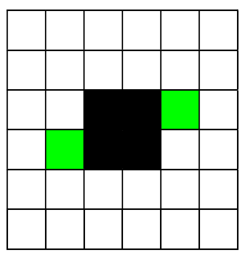
The trick for these problems is to think carefully about each cell one by one.
Let's start with just the dead cells. If it has less than 3 live neighbors, we know it will stay dead. We marked every cell like that yellow in this picture:



Wow! So we know that all of those yellow cells will stay dead at the end of Round 1.

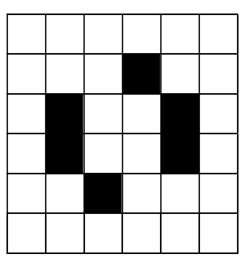
What about the other cells that are dead (the ones that are white above)? Each of them has exactly 3 live neighbors, so they're going to come to life during Round 1! We'll want to remember that.

Now what about the live cells?
These green ones have two or three live neighbors:



That means the green ones will be alive at the end of Round 1! All of the other live cells have four live neighbors, so they will die during Round 1.

Now that we know what happens to every cell, let's put it together! Here's what our grid will look like at the end of Round 1:



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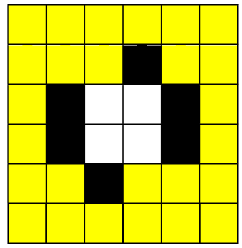
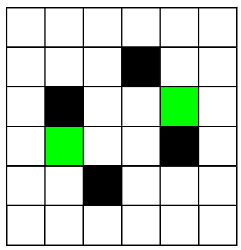
What happens in the 50th Round? First, let's figure out what happens during the 2nd Round.

Just as before, let's mark the dead cells that will stay dead in yellow:



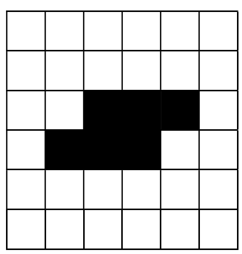
Those center white dead cells each have 3 live neighbors, so they'll come to life during Round 2.

What about the live cells? Well, these green ones have two live neighbors, so they'll stay alive:



The rest have only one live neighbor, and so will die during Round 2.

Putting this together, we get:



But this is just the same as the initial configuration! That means the same cells that died in Round 1 will die again in Round 3, and the same cells that were alive at the end of Round 1 will also be alive at the end of Round 3. In fact, the grid will make the same horizontal design at the end of all odd rounds, and the same vertical design at the end of all even rounds

So after an even number of steps, we always end up with the initial configuration (the horizontal design). 50 is even, so after the 50th Round, we'll end up with the initial configuration.